

14/5/20 - 13:00

Advanced Engine Design (M6X00410)

Iban Adam Hopkins - 1701085

Page 1 of 14

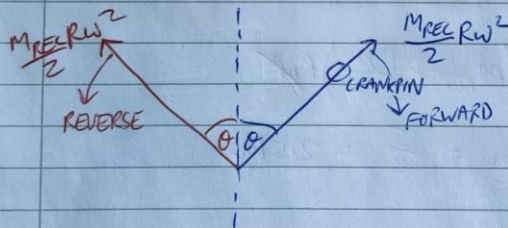
$$① \quad M_{REC} = 0.24 + \frac{1}{3}(0.33) = 0.35 \text{ kg}$$

$$M_{ROT} = \frac{2}{3}(0.33) + 0.02 = 0.24 \text{ kg}$$

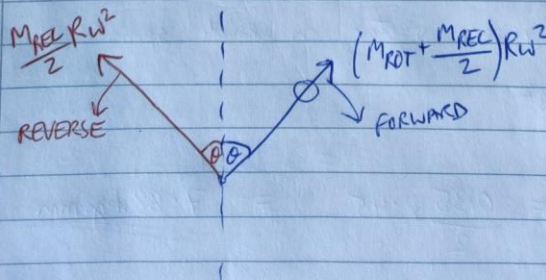
$$R = \frac{90}{2} = 45 \text{ mm}$$

$$L = 140 \text{ mm}$$

- (a) The reciprocating mass results in a first order force of magnitude, $F = M_{REC} \cdot R \omega^2 \cos \theta$, along the cylinder axis. It can be represented by two constant amplitude counter rotating forces as shown below



The rotating mass of the rod creates a centrifugal force acting along the crankpin, resulting in the forces being represented by:



14/5/20 - 13:00

Advanced Engine Design (M6X00410)

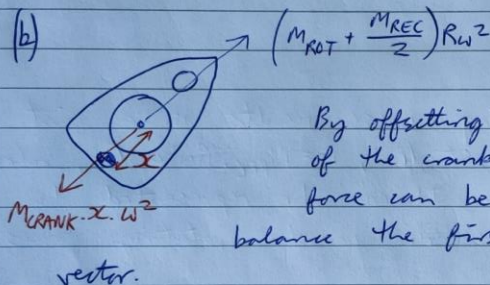
Iban Adam Hopkins - 1701085

Page 2 of 14

First order forward vector on each span

$$= \left(M_{ROT} + \frac{M_{REC}}{2} \right) R \omega^2$$

$$\begin{aligned} \text{Mass moment} &= \left(M_{ROT} + \frac{M_{REC}}{2} \right) R \\ &= \left(0.24 + \frac{0.35}{2} \right) 45 \\ &= \underline{\underline{18.68 \text{ kg} \cdot \text{mm}}} \end{aligned}$$



By offsetting the centre of gravity of the crankshaft, a centrifugal force can be created which will balance the first order forward rotating vector.

To 100% balance, make:

$$M_{CRANK} \cdot x \cdot \omega^2 = \left(M_{ROT} + \frac{M_{REC}}{2} \right) R \omega^2$$

$$\text{Mass of crank span} = \frac{5}{2} = 2.5 \text{ kg}$$

$$2.5 x = 18.68$$

$$\therefore \underline{\underline{x = 7.47 \text{ mm}}}$$

(c) First order reverse vector on each span

$$= \frac{M_{REC}}{2} R \omega^2$$

$$\text{Mass moment} = \frac{0.35}{2} \times 45 = \underline{\underline{7.88 \text{ kg} \cdot \text{mm}}}$$

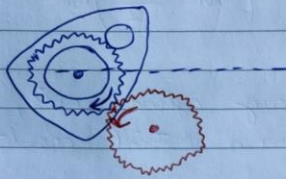
14/5/20 - 13:00

Advanced Engine Design (M6X00410)

ITon Adam Hopkins - 1701085

Page 3 of 14

- (d) The balance shaft would be driven with two equal diameter gears, to rotate the balance shaft counterclockwise at engine speed. Gears would be used as they have less power loss than a belt or a chain, and don't require tensions.

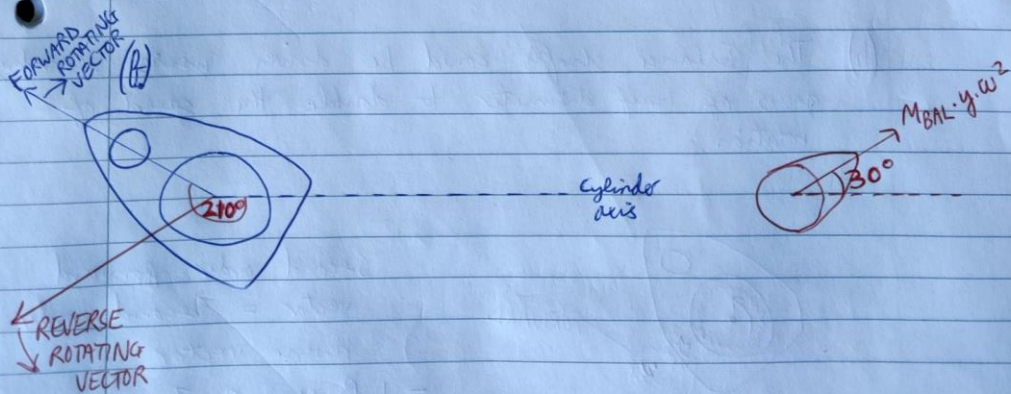


- (e) To fully balance the reverse rotating first order vector make:

$$M_{BAL} \cdot y \cdot \omega^2 = \frac{M_{REL} R \omega^2}{2}$$

$$2.5 y = 7.88$$

$$\therefore y = 2.81 \text{ mm}$$

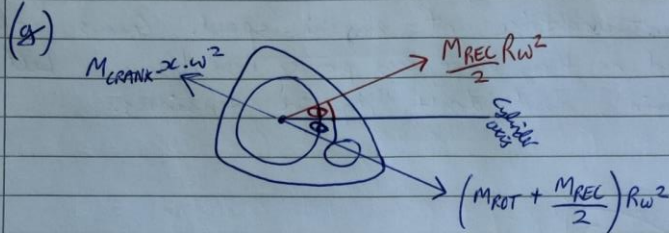


14/5/20 - 13:00

Advanced Engine Design (M6X00410)

Iban Adam Hopkins - 1701085

Page 4 of 14



Make resultant horizontal along cylinder axis, as this will be felt less easily than a vertical resultant.

Make:

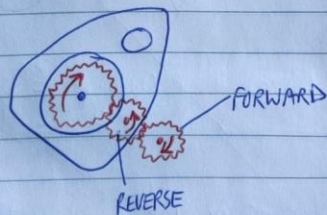
$$\left(M_{ROT} + \frac{M_{REL}}{2}\right)R\omega^2 - M_{CRANK} \cdot x \cdot \omega^2 = \frac{M_{REL} R_w^2}{2}$$

$$M_{CRANK} \cdot x = \left(M_{ROT} + \frac{M_{REL}}{2}\right)R - \frac{M_{REL} R}{2}$$

$$\therefore MM_{CRANK} = 18.68 - 7.88$$

$$MM_{CRANK} = 10.8 \text{ kg.mm}$$

(h) The balance shafts could be driven using secondary gears of half diameter to double the speed of rotation



Second order forward vector = Second order reverse vector

$$= \frac{1}{2} \frac{R}{L} M_{REC} \cdot R \cdot \omega^2$$

14/5/20 - 13:00

Advanced Engine Design (M6X00410)

Iban Adam Hopkins - 1701085

Page 5 of 14

$$\text{Mass moment} = \frac{1}{2} \frac{R}{L} M_{\text{REC}} \cdot R$$

$$\text{Balance force} = M_{\text{BAL}} \cdot y \cdot (2\omega)^2$$

$$= 4 M_{\text{BAL}} \cdot y \cdot \omega^2$$

$$\text{Mass moment} = 4MM$$

for 100% balance make

$$4MM = \frac{1}{2} \frac{R}{L} \cdot M_{\text{REC}} \cdot R$$

$$4MM = \frac{1}{2} \frac{45}{140} \times 0.35 \times 45$$

$$4MM = 2.531$$

$$\therefore \underline{\underline{MM = 0.633 \text{ kg} \cdot \text{mm}}}$$

14/5/20 - 13:00

Advanced Engine Design (M6X00410)

Iban Adam Hopkins - 1701085

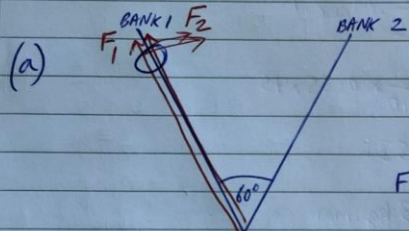
Page 6 of 14

(2) $M_{REL} = 240 + \frac{1}{3}(360) = 360g = 0.36kg$

$$M_{ROT} = \frac{2}{3}(360) = 240g = 0.24kg$$

$$R = \frac{80}{2} = 40mm$$

$$L = 140mm$$



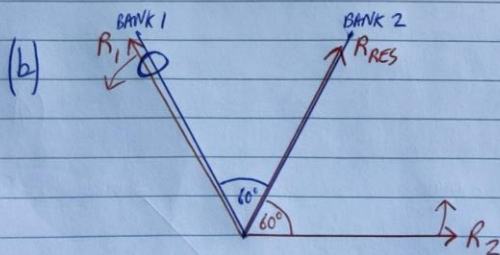
$$F_1 = F_2 = \left(M_{ROT} + \frac{M_{REL}}{2} \right) R \omega^2$$

$$F_{RES} = 2 \left(M_{ROT} + \frac{M_{REL}}{2} \right) R \omega^2$$

Resultant is along bank 1

Resultant mass moment =

$$2 \left(0.24 + \frac{0.36}{2} \right) 40$$
$$= 33.6 \text{ kg}\cdot\text{mm}$$



$$R = R_1 = R_2 = \frac{M_{REL}}{2} R \omega^2$$

$$R_{RES} = R_1 \cos 60 + R_2 \cos 60$$

$$R_{RES} = R \quad (\text{Along bank 2})$$

Resultant First order reverse vector = $\frac{M_{REL}}{2} R \omega^2$

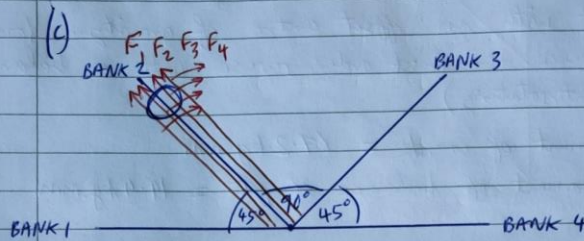
14/5/20 - 13:00

Advanced Engine Design (M6X00410)

Iban Adam Hopkins - 1701085

Page 7 of 14

$$MM = \frac{M_{REC} R}{2} = \frac{0.36 \times 40}{2} = \underline{7.20 \text{ kg} \cdot \text{mm}}$$



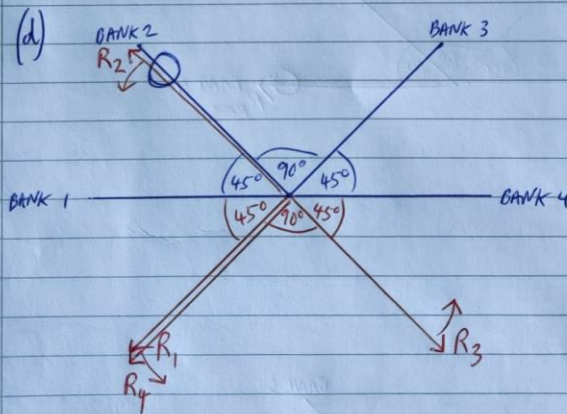
$$F_1 = F_2 = F_3 = F_4 = F$$

$$F = \left(M_{ROT} + \frac{M_{REC}}{2} \right) R \omega^2$$

$$F_{RES} = 4 \left(M_{ROT} + \frac{M_{REC}}{2} \right) R \omega^2 \quad (\text{Along bank 2})$$

$$MM = 4 \left(0.24 + \frac{0.36}{2} \right) 40$$

$$MM = \underline{67.20 \text{ kg} \cdot \text{mm}}$$



$$R_1 = R_2 = R_3 = R_4 = R$$

$$R = \left(\frac{M_{REC}}{2} \right) R \omega^2$$

Resultant first order
reverse vector is
2R opposite bank
3.

$$R_{RES} = 2 \left(\frac{M_{REC}}{2} \right) R \omega^2$$

$$MM = M_{REC} \cdot R = 0.36 \times 40 = \underline{14.40 \text{ kg} \cdot \text{mm}}$$

14/5/20 - 13:00

Advanced Engine Design (M6X00410)

Iban Adam Hopkins - 1701085

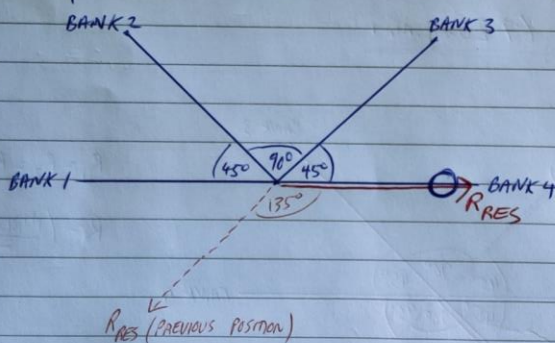
Page 8 of 14

- (c) You would balance the first order forward with a crankshaft with an offset centre of gravity. You would balance the first order reverse with a balance shaft (counterrotating)

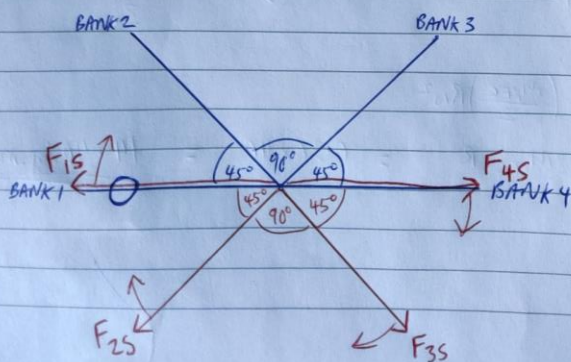
Mass moment of crank = $67.2 \text{ kg} \cdot \text{mm}$

Mass moment of balance shaft = $14.4 \text{ kg} \cdot \text{mm}$

- (d) If the crankpin rotates 135° clockwise from bank 2, then the resultant first order ~~reverse~~ will rotate 135° anticlockwise from its previous position.



(g)



14/5/20 - 13:00

Advanced Engine Design (M6X00410)

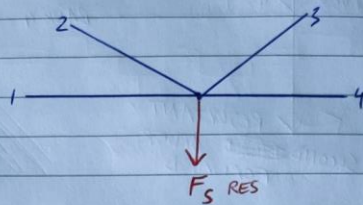
Iban Adam Hopkins - 1701085

Page 9 of 14

$$F_{1s} = F_{2s} = F_{3s} = F_{4s} = F_s$$

$$F_s = \frac{1}{2} \frac{R}{L} M_{REL} \cdot R \omega^2$$

Resultant

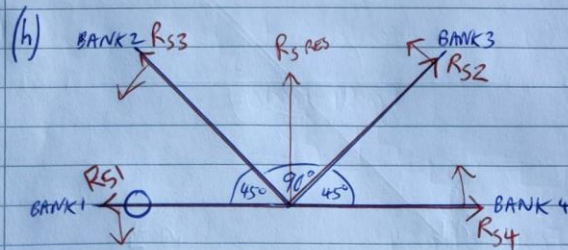


$$F_{sRES} = F_s \cos 45 + F_s \cos 45$$

$$F_{sRES} = F_s \sqrt{2}$$

$$MM = \sqrt{2} \left(\frac{1}{2} \times \frac{40}{140} \times 0.36 \times 40 \right)$$

$$\underline{MM = 2.91 \text{ kg} \cdot \text{mm}}$$



$$R_{s1} = R_{s2} = R_{s3} = R_{s4} = R_s$$

$$R_s = \frac{1}{2} \frac{R}{L} M_{REL} \cdot R \omega^2$$

$$\begin{aligned} \text{Resultant vector} &= R_s \cos 45 + R_s \cos 45 \\ &= R_s \sqrt{2} \end{aligned}$$

$$mm = \sqrt{2} \left(\frac{1}{2} \times \frac{40}{140} \times 0.36 \times 40 \right)$$

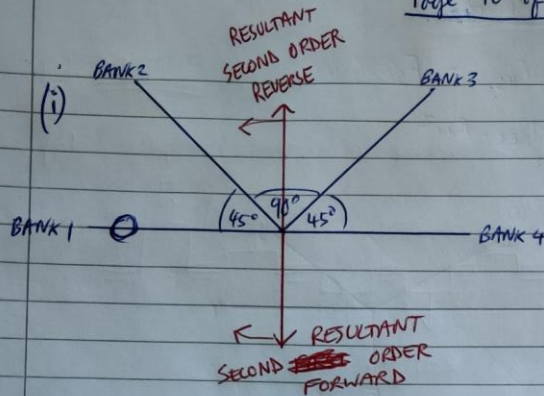
$$\underline{mm = 2.91 \text{ kg} \cdot \text{mm}}$$

14/5/20 - 13:00

Advanced Engine Design (M6X00410)

Iban Adam Hopkins - 1701085

Page 10 of 14



Horizontally, the resultant second orders cancel, as the crankshaft rotates, however, there is a horizontal resultant along bank 1, or bank 4. As the crank rotates the horizontal force will change direction.

14/5/20 - 13:00

Advanced Engine Design (M6X00410)

Iban Adam Hopkins - 1701085

Page 11 of 14

③ $M_{REC} = 240 + \frac{1}{3}(270) = 330 = 0.33 \text{ kg}$

$$M_{ROT} = \frac{2}{3}(270) = 180 = 0.18 \text{ kg}$$

$$R = \frac{85}{2} = 42.5 \text{ mm}$$

$$L = 140 \text{ mm}$$

cylinder centres 92 mm apart

PART 1

(a) $MM = \left(M_{ROT} + \frac{M_{REC}}{2} \right) R$

$$MM = \left(0.18 + \frac{0.33}{2} \right) 42.5$$

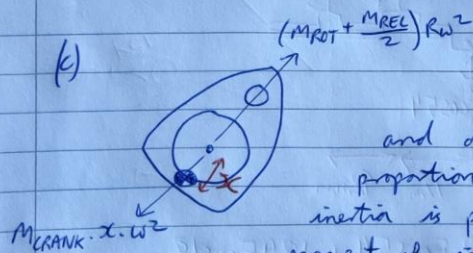
$$MM = \underline{\underline{14.66 \text{ kg} \cdot \text{mm}}}$$

for 100% balance

(b) $M_{CRANK} \cdot x \cdot \omega^2 = \left(M_{ROT} + \frac{M_{REC}}{2} \right) R \omega^2$

$$3. x = 14.66$$

$$x = \underline{\underline{4.89 \text{ mm}}}$$



Tungsten increases mass and decreases x in the same proportion. Because moment of inertia is proportional to $M \cdot x^2$, the moment of inertia decreases if M increases and x decreases in proportion.

This is important in racing as it allows more engine torque to be available to accelerate car.

14/5/20 - 13:00

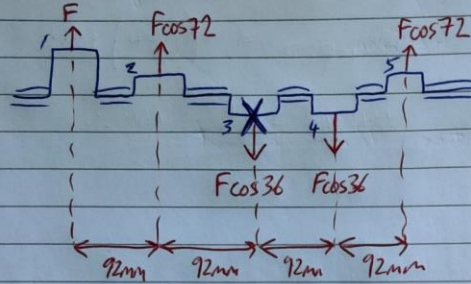
Advanced Engine Design (M6X00410)

Iban Adam Hopkins - 1701085

Page 12 of 14

PART 2

(a) M_{II} - Looking in plane of pin 1



$$F = \left(M_{ROT} + \frac{M_{REC}}{2} \right) R$$

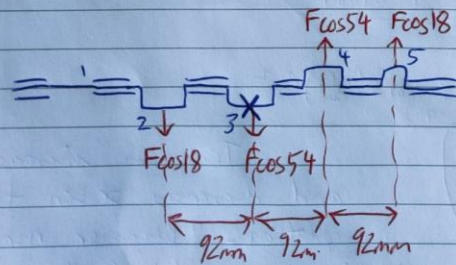
Moments about centre:

$$M_{II} = 184F + 92F \cos 72 + 92F \cos 36 - 184F \cos 72$$

$$M_{II} = 184F + 28.4F + 28.4F - 56.6F$$

$$M_{II} = 184.2F$$

$M_{\perp}$ - Looking perpendicular to plane of pin 1



$$M_{\perp} = 92F \cos 18 + 92F \cos 54 + 184F \cos 18$$

$$M_{\perp} = 87.5F + 54.1F + 175.0F$$

$$M_{\perp} = 316.6F \quad (\text{ANTICLOCKWISE})$$

Cyl 4 and 5 pull rear of crank to the left,

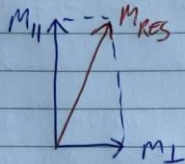
14/5/20 - 13:00

Advanced Engine Design (M6X00410)

Iban Adam Hopkins - 1701085

Page 13 of 14

rotating about X, meaning front of crank is pulled to the right.



$$M_{RES} = \sqrt{M_{II}^2 + M_I^2}$$

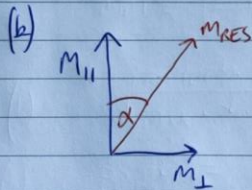
$$M_{RES} = \sqrt{184.2F^2 + 316.6F^2}$$

$$M_{RES} = F \sqrt{184.2^2 + 316.6^2}$$

$$M_{RES} = 366.3F$$

$$M_{RES} = 366.6 \left(M_{ROT} + \frac{M_{REL}}{2} \right) R$$
$$= 366.6 \left(0.18 + \frac{0.33}{2} \right) 42.5$$

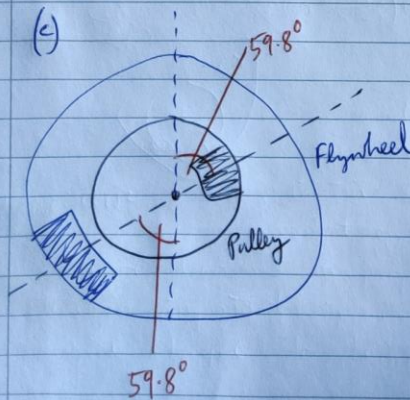
$$\text{Moment of mass moments} = \underline{\underline{5374.4 \text{ kg}\cdot\text{mm}\cdot\text{mm}}}$$



$$\alpha = \tan^{-1} \left(\frac{M_I}{M_{II}} \right)$$

$$\alpha = \tan^{-1} \left(\frac{316.6}{184.2} \right)$$

$$\alpha = \underline{\underline{59.8^\circ}}$$



14/5/20 - 13:00

Advanced Engine Design (M6X00410)

Iban Adam Hopkins - 1701085

Page 14 of 14

$$(d) \quad MM_{\text{flywheel}} = MM_{\text{pulleys}} = MM$$

Counterbalancing moment

$$MM(480) = 5374.4$$

$$\therefore MM = 11.2 \text{ kg}\cdot\text{mm}$$